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Higher level affine Schur and Hecke algebras. **(English summary)**
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In the article under review, the authors complete the construction of a "zoo" of algebras, which consists of two big families, namely,

1. the "Hecke family",
2. the "KLR (short for Khovanov-Lauda-Rouquier) family".

Each of these families is further split into eight sub-families according to being

- **a**ffine or **c**yclotomic,
- **n**o **l**evel or **h**igher **l**evel,
- **S**chur or **n**ot **S**chur.

To efficiently summarize the authors' work, we will use abbreviations. The bulleted properties will be represented by the initials **a**, **c**, **nl**, **hl**, **S** and **nS**. Then, for example, by writing **a.nl.nS.1**, we mean a Hecke algebra that is affine, no level, and not Schur. In this labeling scheme, the main results of this article can be summarized as follows.

First of all, the authors construct the members of the sub-families **a.hl.nS.1**, **c.hl.nS.1** and **a.hl.S.1**. Note that in [Algebr. Comb. **3** (2020), no. 1, 1–38; MR4068742], B. Webster independently constructed the sub-family **a.hl.nS.1**.

Many of these sub-families of algebras are interwoven. Some of these connections were already known before the present article was published. For example, it was shown by J. Brundan and A. S. Kleshchev in [Invent. Math. **178** (2009), no. 3, 451–484; MR2551762], and independently by R. Rouquier in ["2-Kac-Moody algebras", preprint, arXiv:0812.5023], that the blocks of a decomposition of a cyclotomic Hecke algebra are isomorphic to the cyclotomic KLR algebras of type A. Some other coincidences were presented in [B. Webster, op. cit.] and [V. Miemietz and C. Stroppel, Selecta Math. (N.S.) **25** (2019), no. 2, Paper No. 32; MR3948934].

In the present article, the authors construct isomorphisms between suitable completions of the higher level affine Hecke algebras and the higher level KLR algebras. They obtain these isomorphisms as a result of their concrete description of the higher level Hecke algebras in terms of their generators and relations. In fact, the authors identify the natural faithful polynomial representations of both of the completions. Then they show that these polynomial representations agree with each other. Furthermore, the authors show that their isomorphisms descend to give isomorphisms between the cyclotomic versions.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.